HW #5: Solutions

QUESTIONS FOR REVIEW

1. Fill in appropriate terms for the spaces below.

A person who would prefer a certain return of a given amount to a risky investment with the same expected return is (a) risk averse. The maximum amount of money that a (b) risk-averse person would pay to avoid taking a risk is called (c) the risk premium.

Risk can be reduced by (d) diversification, (e) insurance, and (f) additional information.

3. George has $5,000 to invest in a mutual fund. The expected return on mutual fund A is 15% and the expected return on mutual fund B is 10%. Should George pick mutual fund A or fund B?

George’s decision will depend not only on the expected return for each fund, but also on the variability in the expected return on each fund, and on George’s preferences. For example, if fund A has a lower standard deviation than fund B, and he is either risk averse or neutral, then he will choose fund A. If fund A has a higher standard deviation than fund B and he is risk averse, he may choose either A or B or a combination of A and B.

EXERCISES

5. You are an insurance agent who has to write a policy for a new client named Sam. His company, Society for Creative Alternatives to Mayonnaise (SCAM), is working on a low-fat, low-cholesterol mayonnaise substitute for the sandwich condiment industry. The sandwich industry will pay top dollar to whoever invents such a mayonnaise substitute first. Sam’s SCAM seems like a very risky proposition to you. You have calculated his possible returns table as follows.

<table>
<thead>
<tr>
<th>Probability</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>.999</td>
<td>-$1,000,000</td>
</tr>
<tr>
<td>.001</td>
<td>$1,000,000,000</td>
</tr>
</tbody>
</table>

(a) What is the expected return of his project? What is the variance?

The expected return, $ER$, of the investment is

$$ ER = (0.999)(-1,000,000) + (0.001)(1,000,000,000) = 1,000. $$

The variance is

$$ \sigma^2 = (0.999)(-1,000,000 - 1,000)^2 + (0.001)(1,000,000,000 - 1,000)^2, $$

or

$$ \sigma^2 = 1,000,998,999,000,000. $$

(b) What is the most Sam is willing to pay for insurance? Assume Sam is risk neutral.

Because Sam is risk neutral, Sam is unwilling to buy insurance.

(c) Suppose you found out that the Japanese are on the verge of introducing their own mayonnaise substitute next month. Sam does not know this and has just turned down your final offer of $1,000 for the insurance. Assume that Sam tells you SCAM is only six months away from perfecting its mayonnaise substitute and that you know what you know about the Japanese. Would you raise or lower your policy premium on any subsequent proposal to Sam? Based on his information, would Sam accept?
The entry of the Japanese lowers Sam’s probability of a high payoff. For example, assume that the probability of the billion dollar payoff is lowered to zero. Then the expected outcome is:

\[(1.0)(-\$1,000,000) + (0.0)(\$1,000,000,000) = -\$1,000,000.\]

Therefore, you should raise the policy premium substantially. But Sam, not knowing about the Japanese entry, will continue to refuse your offers to insure his losses.

6. Suppose that Natasha’s utility function is given by \( u(I) = \sqrt{10I} \), where \( I \) represents annual income in thousands of dollars.


Natasha is risk averse. To show this, assume that she has $10,000 and is offered a gamble of a $1,000 gain with 50 percent probability and a $1,000 loss with 50 percent probability. Her utility of $10,000 is 10, \( (u(I) = \sqrt{10 \times 10} = 10) \). Her expected utility is:

\[ EU = (0.5)(900.5) + (0.5)(1100.5) = 9.987 < 10. \]

She would avoid the gamble.

b. Suppose that Natasha is currently earning an income of $40,000 (\( I = 40 \)) and can earn that income next year with certainty. She is offered a chance to take a new job that offers a .6 probability of earning $44,000, and a .4 probability of earning $33,000. Should she take the new job?

The utility of her current salary is 4000.5, which is 20. The expected utility of the new job is

\[ EU = (0.6)(4400.5) + (0.4)(3300.5) = 19.85, \]

which is less than 20. Therefore, she should not take the job.

c. In (b), would Natasha be willing to buy insurance to protect against the variable income associated with the new job? If so, how much would she be willing to pay for that insurance? (Hint: What is the risk premium?)

Assuming that she takes the new job and ignoring her current job, Natasha would be willing to pay a risk premium equal to the difference between the expected income of her new job: $39,600 and the certain income which gives her the same level of expected utility of the new job: \( EU = 19.85 \) (see the section b.).

Substituting into her utility function we have, \( 19.85 = (10I)^{0.5} \), and solving for \( I \) we find the income associated with the gamble to be $39,410. Thus, Natasha would be willing to pay for insurance equal to the risk premium, $39,600 - $39,410 = $190.