Unpublished document available at
http://www.res.otaru-uc.ac.jp/~emt-hogasa/,
https://barrel.repo.nii.ac.jp/.
May, 2019

Errata for the paper "A family of the information criteria using the phi-divergence for categorical data" and its supplement

Haruhiko Ogasawara
Otaru University of Commerce

This article gives errata for Ogasawara (2018a) and its supplement (Ogasawara, 2018b) in Sections E1 and E2, respectively. Having corrected the errors, the asymptotic results in the paper have become closer to the corresponding simulated values with increased proportions of choosing correct models by using corrected asymptotic formulas.

References

Ogasawara, H. (2018a). A family of the information criteria using the phi-divergence for categorical data. *Computational Statistics and Data Analysis, 124,* 87-103. https://doi.org/10.1016/j.csda.2018.03.001.

Ogasawara, H. (2018b). An expository supplement to the paper "A family of the information criteria using the phi-divergence for categorical data". *Economic Review* (*Otaru University of Commerce)*, *69* (2 & 3), 11-29. http://www.res.otaru-uc.ac.jp/~emt-hogasa/, http://hdl.handle.net/10252/00005844.

E1. Errata of Ogasawara (2018a)

Page 90, line 7: The term in (2.15)

$$+\frac{1}{2}\left\{\frac{\partial^2 \pi_{0k}}{(\partial \boldsymbol{\theta}_0')^{<2>}}\left(\frac{\partial \boldsymbol{\theta}_0}{\partial \boldsymbol{\pi}_0'}\right)^{<2>} + \frac{\partial \pi_{0k}}{\partial \boldsymbol{\theta}_0'}\frac{\partial^2 \boldsymbol{\theta}_0}{(\partial \boldsymbol{\pi}_0')^{<2>}}\right\} n^2 \mathrm{E}\{(\mathbf{p}-\boldsymbol{\pi}_0)^{<2>}(p_k-\pi_{0k})^2\}_{\to O(n^{-2})}\Bigg]_{\text{(D)}}$$

should be

$$+\frac{1}{2}\left\{\frac{\partial^2\pi_{0k}}{(\partial\boldsymbol{\theta}_0')^{<2>}}\left(\frac{\partial\boldsymbol{\theta}_0}{\partial\boldsymbol{\pi}_0'}\right)^{<2>}+\frac{\partial\pi_{0k}}{\partial\boldsymbol{\theta}_0'}\frac{\partial^2\boldsymbol{\theta}_0}{(\partial\boldsymbol{\pi}_0')^{<2>}}\right\}\Big[n^2\mathrm{E}\{(\mathbf{p}-\boldsymbol{\pi}_0)^{<2>}(p_k-\pi_{0k})^2\}_{\to O(n^{-2})}$$

$$-n\mathrm{E}\{(\mathbf{p}-\boldsymbol{\pi}_0)^{<2>}\}n\mathrm{E}\{(p_k^*-\pi_{0k})^2\}\Big]\Bigg]_{\text{(D)}}.$$

Page 90, line 12: The term in (2.15)

$$+\frac{1}{4\pi_{0k}^3}(\phi_2^{(4)}+4\phi_2^{(3)}+2\phi_2'')\left(\frac{\partial\pi_{0k}}{\partial\boldsymbol{\theta}_0'}\frac{\partial\boldsymbol{\theta}_0}{\partial\boldsymbol{\pi}_0'}\right)^{<2>}n^2\mathrm{E}\{(\mathbf{p}-\boldsymbol{\pi}_0)^{<2>}(p_k-\pi_{0k})^2\}_{\to O(n^{-2})}$$

should be

$$+\frac{1}{4\pi_{0k}^3}(\phi_2^{(4)}+4\phi_2^{(3)}+2\phi_2'')\left(\frac{\partial\pi_{0k}}{\partial\boldsymbol{\theta}_0'}\frac{\partial\boldsymbol{\theta}_0}{\partial\boldsymbol{\pi}_0'}\right)^{<2>}\Big[n^2\mathrm{E}\{(\mathbf{p}-\boldsymbol{\pi}_0)^{<2>}(p_k-\pi_{0k})^2\}_{\to O(n^{-2})}$$

$$-n\mathrm{E}\{(\mathbf{p}-\boldsymbol{\pi}_0)^{<2>}\}n\mathrm{E}\{(p_k^*-\pi_{0k})^2\}\Big].$$

Page 91, (3.1): The equation

$$b_\Delta=\frac{1}{\phi_2''}\left\{-\left(\frac{\phi_2^{(4)}}{2}+2_2^{(3)}+2\phi_2''\right)\sum_{k=1}^{K}\frac{1}{\pi_{0k}}+\phi_2^{(4)}\left(K-\frac{1}{2}\right)+\phi_2^{(3)}(4K-2)+\phi_2''(3K-1)\right\} \quad (3.1)$$

should be

$$b_\Delta=\frac{1}{\phi_2''}\left\{-\left(\phi_2^{(4)}+4\phi_2^{(3)}+3\phi_2''\right)\sum_{k=1}^{K}\frac{1}{\pi_{0k}}+\phi_2^{(4)}\left(2K-1\right)+\phi_2^{(3)}(8K-4)+\phi_2''(5K-2)\right\}. \quad (3.1)$$

Page 91, (3.2): The equation

$$
\begin{aligned}
b_\Delta &= -\frac{1}{2}\{(\lambda-1)(\lambda-2)+4(\lambda-1)+4\}\sum_{k=1}^{K}\frac{1}{\pi_{0k}} \\
&\quad +\frac{1}{2}(\lambda-1)(\lambda-2)(2K-1)+(\lambda-1)(4K-2)+3K-1 \qquad (3.2)\\
&= -\frac{1}{2}(\lambda^2+\lambda+2)\left(\sum_{k=1}^{K}\frac{1}{\pi_{0k}}-2K+1\right)-K+1 \\
&\le -(K-1)\left\{\frac{1}{2}(\lambda^2+\lambda+2)(K-1)+1\right\} \\
&\le -\frac{1}{8}(K-1)(7K+1)
\end{aligned}
$$

should be

$$
\begin{aligned}
b_\Delta &= -\{(\lambda-1)(\lambda-2)+4(\lambda-1)+3\}\sum_{k=1}^{K}\frac{1}{\pi_{0k}} \\
&\quad +(\lambda-1)(\lambda-2)(2K-1)+(\lambda-1)(8K-4)+5K-2 \qquad (3.2)\\
&= -(\lambda^2+\lambda+1)\left(\sum_{k=1}^{K}\frac{1}{\pi_{0k}}-2K+1\right)-K+1 \\
&\le -(K-1)\left\{(\lambda^2+\lambda+1)(K-1)+1\right\} \\
&\le -\frac{1}{4}(K-1)(3K+1).
\end{aligned}
$$

Page 93, lines 32-34: The values "2.09", "0.0209" and "0.8422" (two times) should be "2.08", "0.0208" and "0.8421", respectively.

Page 93, line 12 from below: The term "$-21,\ -31,\ -39,\ -75$ and -30" should be "$-21,\ -41,\ -57,\ -129$ and -39".

Page 101, (A.4): The factor $\frac{2n}{\phi_2''}$ on the right-hand side of the last (not first) equation of

(A.4) should be $\dfrac{2}{\phi_2''}$. Two additional terms should be inserted in $\Big[\ \cdot\ \Big]_{(A)\,(A)}$. That is, the right-hand side of the last equation of (A.4) should be

$$
\begin{aligned}
&= \frac{2}{\phi_2''}\sum_{k=1}^{K}\Bigg[_{(A)} \frac{1}{2}\sum_{\substack{i\neq 0\\ i\neq 2}}^{2}\binom{2}{i}\frac{\partial^2 \hat{\pi}_k\phi_2(p_k/\hat{\pi}_k)}{\partial\hat{\pi}_k^i\partial p_k^{2-i}}\Big|_{\substack{\hat{\pi}_k=\pi_{0k}\\ p_k=\pi_{0k}}} n\mathrm{E}\{(\hat{\pi}_k-\pi_{0k})^i(p_k-\pi_{0k})^{2-i}\}_{\to O(n^{-1})+O(n^{-2})}\\
&+\frac{n^{-1}}{6}\sum_{\substack{i\neq 0\\ i\neq 3}}^{3}\binom{3}{i}\frac{\partial^3 \hat{\pi}_k\phi_2(p_k/\hat{\pi}_k)}{\partial\hat{\pi}_k^i\partial p_k^{3-i}}\Big|_{\substack{\hat{\pi}_k=\pi_{0k}\\ p_k=\pi_{0k}}} n^2\mathrm{E}\{(\hat{\pi}_k-\pi_{0k})^i(p_k-\pi_{0k})^{3-i}\}_{\to O(n^{-2})}\\
&+\frac{n^{-1}}{24}\sum_{\substack{i\neq 0\\ i\neq 4}}^{4}\binom{4}{i}\frac{\partial^4 \hat{\pi}_k\phi_2(p_k/\hat{\pi}_k)}{\partial\hat{\pi}_k^i\partial p_k^{4-i}}\Big|_{\substack{\hat{\pi}_k=\pi_{0k}\\ p_k=\pi_{0k}}} n^2\mathrm{E}\{(\hat{\pi}_k-\pi_{0k})^i(p_k-\pi_{0k})^{4-i}\}_{\to O(n^{-2})}\\
&-\frac{n^{-1}}{6}\binom{3}{1}\frac{\partial^3 \hat{\pi}_k\phi_2(p_k^*/\hat{\pi}_k)}{\partial\hat{\pi}_k\partial p_k^{*2}}\Big|_{\substack{\hat{\pi}_k=\pi_{0k}\\ p_k^*=\pi_{0k}}} n\mathrm{E}(\hat{\pi}_k-\pi_{0k})_{\to O(n^{-1})}n\mathrm{E}\{(p_k^*-\pi_{0k})^2\}\\
&-\frac{n^{-1}}{24}\binom{4}{2}\frac{\partial^4 \hat{\pi}_k\phi_2(p_k^*/\hat{\pi}_k)}{\partial\hat{\pi}_k^2\partial p_k^{*2}}\Big|_{\substack{\hat{\pi}_k=\pi_{0k}\\ p_k^*=\pi_{0k}}} n\mathrm{E}\{(\hat{\pi}_k-\pi_{0k})^2\}_{\to O(n^{-1})}n\mathrm{E}\{(p_k^*-\pi_{0k})^2\}\ \Bigg]_{(A)}+O(n^{-2})
\end{aligned}
$$

Page 102, (A.9): In the second term of the second line on the right-hand side of the first equation of (A.9), the factor "(2/3)" should be inserted. The factor "(3/2)" on the right-hand side of the second equation of (A.9) should be deleted. The factor $\left(\dfrac{1}{2}\phi_2^{(4)}+2\phi_2^{(3)}+3\phi_2''\right)$ on the right-hand side of the last equation of (A.9) should be $(\phi_2^{(4)}+4\phi_2^{(3)}+4\phi_2'')$. That is, (A.9) should be

$$
\begin{aligned}
b_\Delta = \frac{2}{\phi_2''}\sum_{k=1}^{K}\Bigg\{&-\frac{1}{2\pi_{0k}^2}(\phi_2^{(3)}+\phi_2'')\kappa_3(k)+\frac{1}{2\pi_{0k}^2}(\phi_2^{(3)}+2\phi_2'')\kappa_3(k)\\
&-\frac{1}{6\pi_{0k}^3}(\phi_2^{(4)}+2\phi_2^{(3)})m_4(k)+\frac{1}{4\pi_{0k}^3}(\phi_2^{(4)}+4\phi_2^{(3)}+2\phi_2'')(2/3)m_4(k)\\
&-\frac{1}{6\pi_{0k}^3}(\phi_2^{(4)}+6\phi_2^{(3)}+6\phi_2'')m_4(k)\Bigg\}
\end{aligned}
\qquad \text{(A.9)}
$$

$$
\begin{aligned}
&= \frac{1}{\phi_2''} \sum_{k=1}^{K} \Bigg[\{-(\phi_2^{(3)} + \phi_2'') + \phi_2^{(3)} + 2\phi_2''\} \left(\frac{1}{\pi_{0k}} - 3 + 2\pi_{0k} \right) \\
&\quad + \{-(\phi_2^{(4)} + 2\phi_2^{(3)}) + \phi_2^{(4)} + 4\phi_2^{(3)} + 2\phi_2'' - (\phi_2^{(4)} + 6\phi_2^{(3)} + 6\phi_2'')\} \\
&\quad \times \left(\frac{1}{\pi_{0k}} - 2 + \pi_{0k} \right) \Bigg] \\
&= \frac{1}{\phi_2''} \sum_{k=1}^{K} \left\{ \phi_2'' \left(\frac{1}{\pi_{0k}} - 3 + 2\pi_{0k} \right) - \left(\phi_2^{(4)} + 4\phi_2^{(3)} + 4\phi_2'' \right) \left(\frac{1}{\pi_{0k}} - 2 + \pi_{0k} \right) \right\}
\end{aligned}
$$

Tables 3, 4, 6 and 8 should be as follows:

Table 3 (Corrected after publication; May 16, 2019). Added percentages of the correct (simplest) models selected by the $M^*\phi IC$ s over those by the ϕIC s (the number of replications = 10,000)

	The genetics of plants (Fisher, 1970; 4 categories)			3-category truncated Poisson (Bishop et al., 1975)			4-category truncated Poisson		
n:	50	200	800	50	200	800	50	200	800
The ϕ_2 in a ϕIC matches the ϕ_1 for estimation of parameters.									
$\lambda = 0$ (G^2)	2.08	.38	.13	.39	.59	.05	1.39	.31	.07
$\lambda = -1$ (GM^2)	1.72	.41	.17	.46	.15	.01	1.38	.33	.03
$\lambda = -2$ (Neyman)	5.01	1.42	.27	.69	.45	.03	3.79	.80	.22
$\lambda = -0.5$ (T^2)	1.63	.45	.09	.33	.34	.08	1.70	.17	.07
$\lambda = 0.5$	3.21	1.14	.26	.34	.23	.12	2.41	.54	.12
$\lambda = 2/3$ (C-R)	4.53	1.26	.41	.45	.20	.16	2.55	.73	.11
$\lambda = 1$ (X^2)	5.68	1.45	.41	1.10	.18	.37	3.59	.90	.23
$\lambda = 2$	8.49	3.18	.85	3.19	.58	.05	6.78	1.84	.44
GJ^2	2.25	.56	.12	.46	.35	.08	1.86	.28	.10
E_g^2	3.67	1.13	.24	.96	.18	.37	2.20	.65	.14
NX^2	5.71	1.24	.44	.75	.89	.09	3.97	.80	.31
The MLEs ($\lambda = 0$) are used for all power divergences.									
$\lambda = 0$ (G^2)	2.36	.40	.22	.48	.53	.05	1.72	.37	.08
$\lambda = -1$ (GM^2)	2.70	.46	.18	.44	.13	.00	1.75	.33	.07
$\lambda = -2$ (Neyman)	6.04	1.44	.34	.72	.47	.03	3.46	.85	.22
$\lambda = -0.5$ (T^2)	2.64	.37	.07	.46	.37	.04	1.47	.32	.13
$\lambda = 0.5$	3.35	.78	.25	.40	.27	.11	2.63	.45	.11
$\lambda = 2/3$ (C-R)	5.01	.98	.19	.56	.25	.10	2.80	.54	.14
$\lambda = 1$ (X^2)	6.39	1.37	.33	1.05	.24	.21	3.85	.81	.20
$\lambda = 2$	11.64	3.24	.79	3.38	.61	.06	7.13	1.84	.35
GJ^2	3.18	.49	.12	.46	.37	.04	1.78	.42	.14
E_g^2	5.39	1.01	.22	.98	.24	.21	2.42	.56	.14
NX^2	7.18	1.21	.36	.88	.83	.04	3.91	1.03	.33

Table 3 (Corrected after publication; May 16, 2019). (continued)

	The genetics of plants (Fisher, 1970; 4 categories)			3-category truncated Poisson (Bishop et al., 1975)			4-category truncated Poisson		
n:	50	200	800	50	200	800	50	200	800
The parameter estimators by $\lambda = 1$ (X^2) are used for all power divergences.									
$\lambda = 0$ (G^2)	2.77	.59	.12	.51	.54	.15	1.36	.31	.09
$\lambda = -1$ (GM^2)	2.03	.49	.06	.51	.09	.01	1.62	.20	.02
$\lambda = -2$ (Neyman)	5.10	1.48	.35	.78	.35	.04	3.77	.76	.20
$\lambda = -0.5$ (T^2)	1.64	.39	.14	.51	.31	.08	1.62	.27	.08
$\lambda = 0.5$	3.37	.86	.25	.41	.24	.17	2.39	.54	.16
$\lambda = 2/3$ (C-R)	4.91	1.13	.29	.57	.23	.14	2.42	.61	.16
$\lambda = 1$ (X^2)	6.04	1.39	.30	1.12	.22	.27	3.67	.71	.19
$\lambda = 2$	8.68	3.33	.82	3.05	.54	.04	6.37	1.85	.55
GJ^2	2.34	.57	.20	.51	.31	.08	1.73	.29	.08
E_g^2	3.99	1.10	.16	.97	.22	.26	2.29	.55	.13
NX^2	5.08	1.47	.41	.80	.65	.10	3.53	.85	.17
The parameter estimators by E_g^2 (Eguchi) are used for all power divergences.									
$\lambda = 0$ (G^2)	2.54	.51	.11	.48	.55	.03	1.69	.31	.03
$\lambda = -1$ (GM^2)	1.93	.50	.15	.44	.12	.01	1.54	.26	.04
$\lambda = -2$ (Neyman)	5.25	1.54	.26	.81	.32	.05	3.35	.86	.19
$\lambda = -0.5$ (T^2)	1.71	.33	.16	.45	.42	.05	1.64	.27	.06
$\lambda = 0.5$	3.34	1.02	.29	.42	.25	.21	2.48	.55	.09
$\lambda = 2/3$ (C-R)	4.68	1.21	.35	.57	.24	.19	2.70	.64	.11
$\lambda = 1$ (X^2)	6.19	1.36	.38	1.05	.21	.24	4.02	.77	.14
$\lambda = 2$	8.90	3.39	.76	3.32	.56	.09	7.17	1.64	.49
GJ^2	2.63	.46	.19	.45	.42	.05	1.79	.34	.09
E_g^2	4.06	1.04	.23	.92	.21	.24	2.36	.55	.12
NX^2	4.96	1.23	.35	.93	.80	.14	3.68	.75	.23

Note. n = the number of observations, Z = the number of deleted cases with zero frequenc(ies), NC = the number of deleted cases due to non-convergence, G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, T^2 = the Freeman-Tukey statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic, GJ^2 = Jeffreys' divergence, E_g^2 = Eguchi's divergence, NX^2 = (Neyman's statistic + X^2)/2.

Table 4 (Corrected after publication for H.A.B., b_Δ and b_Δ^* except the case of $\lambda = 0$ (G^2); May 16, 2019). Simulated and asymptotic biases of the power divergences (the number of replications = 10,000)

The ϕ_2 in a ϕIC matches the ϕ_1 for estimation of parameters.

The genetics of plants (Fisher, 1970; 4 categories)

Model 1	S.B.	A.B.	H.A.B.	S.b_Δ	b_Δ	S.B.	A.B.	H.A.B.	S.b_Δ	b_Δ
			$n = 50$					$n = 200$		
$\lambda = 0$ (G^2)	-2.48	-2	-2.13	-23.9	-6.6	-2.08	-2	-2.03	-15.4	-6.6
$\lambda = 2/3$ (C-R)	-2.61	-2	-2.22	-30.6	-10.8	-2.10	-2	-2.05	-20.1	-10.8
$\lambda = 1$ (X^2)	-2.78	-2	-2.32	-39.0	-15.9	-2.13	-2	-2.08	-25.8	-15.9
$\lambda = 2$	-3.78	-2	-2.88	-89.1	-43.8	-2.28	-2	-2.22	-56.9	-43.8
E_g^2	-2.57	-2	-2.18	-28.5	-9.2	-2.09	-2	-2.05	-18.4	-9.2
			$n = 800$			(b_Δ^*)				
$\lambda = 0$ (G^2)	-2.00	-2	-2.01	1.7	-6.6	(-7.0)				
$\lambda = 2/3$ (C-R)	-2.00	-2	-2.01	-.2	-10.8	(-13.7)				
$\lambda = 1$ (X^2)	-2.01	-2	-2.02	-4.3	-15.9	(-19.0)				
$\lambda = 2$	-2.04	-2	-2.05	-29.0	-43.8	(-43.0)				
E_g^2	-2.00	-2	-2.01	2.4	-9.2	(-13.0)				
Model 2			$n = 50$					$n = 200$		
$\lambda = 0$ (G^2)	-4.82	-4	-4.24	-40.9	-12.0	-4.11	-4	-4.06	-22.1	-12.0
$\lambda = 2/3$ (C-R)	-3.94	-4	-4.47	3.0	-23.5	-4.08	-4	-4.12	-16.6	-23.5
$\lambda = 1$ (X^2)	-4.04	-4	-4.68	-2.0	-34.0	-4.13	-4	-4.17	-26.2	-34.0
$\lambda = 2$	-4.62	-4	-5.69	-31.2	-84.7	-4.26	-4	-4.42	-51.1	-84.7
E_g^2	-3.82	-4	-4.43	9.2	-21.7	-3.96	-4	4.11	7.7	-21.7
			$n = 800$			(b_Δ^*)				
$\lambda = 0$ (G^2)	-4.06	-4	-4.02	-46.4	-12.0	(-14.0)				
$\lambda = 2/3$ (C-R)	-4.07	-4	-4.03	-55.2	-23.5	(-27.3)				
$\lambda = 1$ (X^2)	-4.08	-4	-4.04	-64.7	-34.0	(-38.0)				
$\lambda = 2$	-4.14	-4	-4.11	-114.1	-84.7	(-86.0)				
E_g^2	-4.06	-4	-4.03	-51.6	-21.7	(-26.0)				

Table 4 (Corrected after publication for H.A.B., b_Δ and b_Δ^* except the case of $\lambda = 0$ (G^2); May 16, 2019). (continued)

The ϕ_2 in a ϕIC matches the ϕ_1 for estimation of parameters.										
The genetics of plants (Fisher, 1970; 4 categories)										
Model 3	S.B.	A.B.	H.A.B.	S. b_Δ	b_Δ	S.B.	A.B.	H.A.B.	S. b_Δ	b_Δ
			$n = 50$					$n = 200$		
$\lambda = 0$ (G^2)	-6.03	-6	-6.42	-1.6	-21.0	-6.01	-6	-6.11	-2.0	-21.0
$\lambda = 2/3$ (C-R)	-6.36	-6	-6.82	-17.8	-41.0	-6.09	-6	-6.21	-17.9	-41.0
$\lambda = 1$ (X^2)	-6.70	-6	-7.14	-34.9	-57.0	-6.16	-6	-6.29	-32.4	-57.0
$\lambda = 2$	-8.99	-6	-8.58	-149.3	-129.0	-6.53	-6	-6.65	-105.6	-129.0
E_g^2	-6.21	-6	-6.78	-10.7	-39.0	-6.07	-6	-6.20	-13.1	-39.0
			$n = 800$			($b_\Delta^* = b_\Delta$)				
$\lambda = 0$ (G^2)	-6.10	-6	-6.03	-80.5	-21.0	(-21.0)				
$\lambda = 2/3$ (C-R)	-6.12	-6	-6.05	-97.5	-41.0	(-41.0)				
$\lambda = 1$ (X^2)	-6.14	-6	-6.07	-112.5	-57.0	(-57.0)				
$\lambda = 2$	-6.23	-6	-6.16	-184.4	-129.0	(-129.0)				
E_g^2	-6.12	-6	-6.05	-93.1	-39.0	(-39.0)				

Note. n = the number of observations, S.B. = simulated bias, A.B. = asymptotic bias $= -2q$, H.A.B. = $b + n^{-1}b_\Delta = -2q + n^{-1}b_\Delta$, S. b_Δ = simulated $b_\Delta = n(\text{S.B.} + 2q)$, G^2 = the log-likelihood ratio statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic, E_g^2 = Eguchi's divergence. The number for model identification is the number of independent parameters.

Table 6 (Corrected after publication for M*ϕIC except the cases of $\lambda = 0$ (G^2), $\lambda = -1$ (GM^2), and GJ^2; May 16, 2019). ϕICs and M*ϕICs of the log-linear models for the $2 \times 2 \times 2$ table on a survey about alcohol, cigarette and marijuana use for US high school seniors

The ϕ_2 in a ϕIC matches the ϕ_1 for estimation of parameters.

Model:	(A, C, M)	(A, CM)	(C, AM)	(M, AC)	(AC, AM)	(AC, CM)	(AM, CM)	(AC, AM, CM)	(ACM)
					ϕIC				
$\lambda = 0$ (G^2)	1292.0	542.2	947.6	851.8	507.4	102.0	197.8	12.37	14
$\lambda = -1$ (GM^2)	1267.2	576.1	1111.8	1178.3	577.6	109.1	215.3	12.35	14
$\lambda = -2$ (Neyman)	**	*	*	*	84.3	206.0	12.32	14	
$\lambda = -0.5$ (T^2)	1328.3	603.6	1058.7	1009.1	553.0	111.3	208.3	12.36	14
$\lambda = 0.5$	1233.9	482.8	843.1	749.2	461.7	89.2	186.0	12.38	14
$\lambda = 2/3$ (C-R)	1219.3	466.9	813.9	724.1	447.9	85.1	182.0	12.39	14
$\lambda = 1$ (X^2)	1200.8	440.3	764.3	684.0	423.1	77.8	174.2	12.39	14
$\lambda = 2$	1234.2	390.5	674.2	619.5	370.5	61.7	152.8	12.41	14
GJ^2	1420.6	662.7	1103.8	1081.6	566.8	116.0	211.2	12.36	14
E_g^2	918.3	279.2	622.0	500.0	350.5	52.8	157.1	12.39	14
NX^2	2107.2	1168.8	1473.9	1584.9	667.7	156.4	233.0	12.37	14
					M*ϕIC				
$\lambda = 0$ (G^2)	1292.4	542.7	948.1	852.3	508.0	102.7	198.4	13.13	14.88
$\lambda = -1$ (GM^2)	1267.6	576.6	1112.3	1178.8	578.2	109.7	215.9	13.11	14.88
$\lambda = -2$ (Neyman)	*	*	*	*	*	86.2	207.9	14.59	16.65
$\lambda = -0.5$ (T^2)	1328.6	604.0	1059.1	1009.5	553.5	111.7	208.7	12.93	14.66
$\lambda = 0.5$	1234.5	483.7	844.0	750.1	462.8	90.3	187.1	13.71	15.55
$\lambda = 2/3$ (C-R)	1220.0	468.0	814.9	725.1	449.3	86.4	183.4	13.99	15.86
$\lambda = 1$ (X^2)	1202.0	441.8	765.8	685.5	425.0	79.6	176.1	14.66	16.65
$\lambda = 2$	1236.9	394.0	677.7	623.0	374.9	66.1	157.2	17.70	20.17
GJ^2	1421.0	663.2	1104.3	1082.1	567.4	116.6	211.8	13.12	14.88
E_g^2	919.0	280.2	623.1	500.8	351.8	54.0	158.4	13.91	15.77
NX^2	2108.4	1170.3	1475.5	1586.5	669.6	158.2	234.9	14.64	16.65

Note. G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, T^2 = the Freeman-Tukey statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic, GJ^2 = Jeffreys' divergence, E_g^2 = Eguchi's divergence, NX^2 = (Neyman's statistic + X^2)/2. An asterisk indicates a non-convergent case.

Table 8 (Corrected after publication for M*ϕIC except the cases of $\lambda = 0$ (G^2), $\lambda = -1$ (GM^2), and GJ^2; May 16, 2019). ϕICs and M*ϕICs of the log-linear models for the 2-way contingency table on opinions about premarital sex and availability of teenage birth control

The ϕ_2 in a ϕIC matches the ϕ_1 for estimation of parameters.				
		ϕIC		
Model:	(X, Y)	(X, Y, linA)	(X, Y, linB)	(X, Y, XY)
$\lambda = 0$ (G^2)	139.7	25.53	22.85	30
$\lambda = -1$ (GM^2)	140.2	25.58	22.87	30
$\lambda = -2$ (Neyman)	137.4	25.57	22.86	30
$\lambda = -0.5$ (T^2)	140.3	25.56	22.86	30
$\lambda = 0.5$	138.6	25.50	22.82	30
$\lambda = 2/3$ (C-R)	138.2	25.48	22.81	30
$\lambda = 1$ (X^2)	137.5	25.45	22.78	30
$\lambda = 2$	136.6	25.34	22.69	30
GJ^2	141.1	25.57	22.87	30
E_g^2	134.8	25.41	22.76	30
NX^2	147.5	25.66	22.92	30
		M*ϕIC		
$\lambda = 0$ (G^2)	139.8	25.73	23.04	30.41
$\lambda = -1$ (GM^2)	140.4	25.77	23.06	30.41
$\lambda = -2$ (Neyman)	137.8	26.13	23.43	31.21
$\lambda = -0.5$ (T^2)	140.5	25.71	23.01	30.31
$\lambda = 0.5$	138.9	25.83	23.15	30.71
$\lambda = 2/3$ (C-R)	138.6	25.88	23.21	30.86
$\lambda = 1$ (X^2)	138.0	26.01	23.35	31.21
$\lambda = 2$	137.7	26.64	24.00	32.80
GJ^2	141.3	25.77	23.06	30.41
E_g^2	135.1	25.79	23.14	30.81
NX^2	147.9	26.22	23.49	31.21

Note. G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, T^2 = the Freeman-Tukey statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic, GJ^2 = Jeffreys' divergence, E_g^2 = Eguchi's divergence, NX^2 = (Neyman's statistic + X^2)/2. In "linA", $\mathbf{u} = (1, 2, 3, 4)'$ and $\mathbf{v} = (1, 2, 3, 4)'$, and in "linB", $\mathbf{u} = (1, 2, 4, 5)'$ and $\mathbf{v} = (1, 2, 4, 5)'$ before standardization.

E2. Errata for Ogasawara (2018b)

Page 12, line 2: A comma should be inserted. That is, the right-hand side of the third equation of (S.1)

$$=\{\mathrm{diag}(\boldsymbol{\pi}_{0(K-1)})-\boldsymbol{\pi}_{0(K-1)}\boldsymbol{\pi}_{0(K-1)}'\}\{\mathrm{diag}^{-1}(\boldsymbol{\pi}_{0(K-1)})-\pi_{0K}^{-1}\mathbf{1}_{(K-1)}\}$$

should be

$$=\{\mathrm{diag}(\boldsymbol{\pi}_{0(K-1)})-\boldsymbol{\pi}_{0(K-1)}\boldsymbol{\pi}_{0(K-1)}'\}\{\mathrm{diag}^{-1}(\boldsymbol{\pi}_{0(K-1)}),-\pi_{0K}^{-1}\mathbf{1}_{(K-1)}\}\,.$$

Page 18, (S.12): The symbol $\sum_{(i,j,k)}^{2}$ on the right-hand side of the first equation of (S.12) should be $\sum_{(i,j,k)}^{3}$.

Page 19-20, (S.14): The factor $m_4(a,b,k,k)$ (two times) in (S.14) should be $\{m_4(a,b,k,k)-(\delta_{ab}\pi_{aa}-\pi_{0a}\pi_{0b})(\pi_{0k}-\pi_{0k}^2)\}$. That is, (S.14) should be

$$\begin{aligned}
b_{\Delta}=\frac{2}{\phi_2''}\sum_{k=1}^{K}\Bigg[_{(\mathrm{A})}\; & -\frac{\phi_2''}{\pi_{0k}}\Bigg[_{(\mathrm{B})}\;\frac{1}{2}\sum_{a,b=1}^{K}(-\delta_{ka}-\delta_{kb}+2\pi_{0k})\kappa_3(a,b,k)\\
& +\frac{1}{6}\sum_{a,b,c=1}^{K}\{2(\mathbf{I}_{(K)})_{ka}+2(\mathbf{I}_{(K)})_{kb}+2(\mathbf{I}_{(K)})_{kc}-6\pi_{0k}\}\; m_4(a,b,c,k)\;\Bigg]_{(\mathrm{B})}\\
& -\frac{1}{2\pi_{0k}^2}(\phi_2^{(3)}+\phi_2'')\Bigg[\sum_{a=1}^{K}(\delta_{ka}-\pi_{0k})\kappa_3(a,k,k)\\
& \qquad +\frac{1}{2}\sum_{a,b=1}^{K}(-\delta_{ka}-\delta_{kb}+2\pi_{0k})\{m_4(a,b,k,k)\\
& \qquad\qquad -(\delta_{ab}\pi_{aa}-\pi_{0a}\pi_{0b})(\pi_{0k}-\pi_{0k}^2)\}\Bigg]
\end{aligned}\qquad (\mathrm{S}.14)$$

$$
\begin{aligned}
&+\frac{1}{2\pi_{0k}^{2}}(\phi_2^{(3)}+2\phi_2'')\sum_{a,b=1}^{K}\{(\delta_{ka}-\pi_{0k})(\delta_{kb}-\pi_{0k})\kappa_3(a,b,k) \\
&\qquad\qquad +\sum_{c=1}^{K}(\delta_{ka}-\pi_{0k})(-\delta_{kb}-\delta_{kc}+2\pi_{0k})m_4(a,b,c,k)\} \\
&-\frac{1}{6\pi_{0k}^{3}}(\phi_2^{(4)}+2\phi_2^{(3)})\sum_{a,b=1}^{K}(\delta_{ka}-\pi_{0k})m_4(a,k,k,k) \\
&+\frac{1}{4\pi_{0k}^{3}}(\phi_2^{(4)}+4\phi_2^{(3)}+2\phi_2'')\sum_{a,b=1}^{K}(\delta_{ka}-\pi_{0k})(\delta_{kb}-\pi_{0k}) \\
&\qquad\qquad \times\{m_4(a,b,k,k)-(\delta_{ab}\pi_{aa}-\pi_{0a}\pi_{0b})(\pi_{0k}-\pi_{0k}^{2})\} \\
&-\frac{1}{6\pi_{0k}^{3}}(\phi_2^{(4)}+6\phi_2^{(3)}+6\phi_2'') \\
&\qquad \times\sum_{a,b,c=1}^{K}(\delta_{ka}-\pi_{0k})(\delta_{kb}-\pi_{0k})(\delta_{kc}-\pi_{0k})m_4(a,b,c,k) \Bigg]. \qquad \text{(A)}
\end{aligned}
$$

Tables S1 to S5 should be as follows.

Table S1 (Corrected after publication for H.A.B., b_Δ and b_Δ^* except in the case of $\lambda = 0$ (G^2); May 16, 2019). Simulated and asymptotic biases of the power divergences (the number of replications = 10,000)

The MLEs ($\lambda = 0$) are used for all power divergences.

The genetics of plants (Fisher, 1970; 4 categories)

Model 1	S.B.	A.B.	H.A.B.	S. b_Δ	b_Δ	S.B.	A.B.	H.A.B.	S. b_Δ	b_Δ
			$n = 50$					$n = 200$		
$\lambda = 0$ (G^2)	-2.06	-2	-2.13	-2.9	-6.6	-2.05	-2	-2.03	-9.7	-6.6
$\lambda = 2/3$ (C-R)	-2.19	-2	-2.23	-9.3	-11.3	-2.08	-2	-2.06	-15.4	-11.3
$\lambda = 1$ (X^2)	-2.31	-2	-2.32	-15.5	-15.9	-2.10	-2	-2.08	-20.7	-15.9
$\lambda = 2$	-3.03	-2	-2.77	-51.3	-38.4	-2.24	-2	-2.19	-47.3	-38.4
E_g^2	-2.14	-2	-2.18	-7.0	-9.2	-2.07	-2	-2.05	-13.4	-9.2
			$n = 800$			(b_Δ^*)				
$\lambda = 0$ (G^2)	-1.91	-2	-2.01	74.4	-6.6	(-7.0)				
$\lambda = 2/3$ (C-R)	-1.92	-2	-2.01	67.4	-11.3	(-13.7)				
$\lambda = 1$ (X^2)	-1.92	-2	-2.02	61.8	-15.9	(-19.0)				
$\lambda = 2$	-1.96	-2	-2.05	35.8	-38.4	(-43.0)				
E_g^2	-1.91	-2	-2.01	68.4	-9.2	(-13.0)				
Model 2			$n = 50$					$n = 200$		
$\lambda = 0$ (G^2)	-4.17	-4	-4.24	-8.4	-12.0	-4.07	-4	-4.06	-13.2	-12.0
$\lambda = 2/3$ (C-R)	-4.49	-4	-4.48	-24.7	-24.1	-4.14	-4	-4.12	-27.3	-24.1
$\lambda = 1$ (X^2)	-4.81	-4	-4.69	-40.3	-34.3	-4.20	-4	-4.17	-39.2	-34.3
$\lambda = 2$	-6.84	-4	-5.63	-142.2	-81.3	-4.49	-4	-4.41	-98.4	-81.3
E_g^2	-4.42	-4	-4.44	-20.9	-22.0	-4.12	-4	-4.11	-24.8	-22.0
			$n = 800$			(b_Δ^*)				
$\lambda = 0$ (G^2)	-3.95	-4	-4.02	42.4	-12.0	(-14.0)				
$\lambda = 2/3$ (C-R)	-3.97	-4	-4.03	26.5	-24.1	(-27.3)				
$\lambda = 1$ (X^2)	-3.98	-4	-4.04	14.3	-34.3	(-38.0)				
$\lambda = 2$	-4.05	-4	-4.10	-40.0	-81.3	(-86.0)				
E_g^2	-3.97	-4	-4.03	27.0	-22.0	(-26.0)				

Table S1 (Corrected after publication for H.A.B., b_Δ and b_Δ^* except the case of $\lambda = 0$ (G^2); May 16, 2019). (continued)

The MLEs ($\lambda = 0$) are used for all power divergences.										
The genetics of plants (Fisher, 1970; 4 categories)										
Model 3	S.B.	A.B.	H.A.B.	S. b_Δ	b_Δ	S.B.	A.B.	H.A.B.	S. b_Δ	b_Δ
			$n = 50$					$n = 200$		
$\lambda = 0$ (G^2)	-6.39	-6	-6.42	-19.7	-21.0	-6.12	-6	-6.11	-24.0	-21.0
$\lambda = 2/3$ (C-R)	-6.99	-6	-6.82	-49.5	-41.0	-6.24	-6	-6.21	-47.2	-41.0
$\lambda = 1$ (X^2)	-7.54	-6	-7.14	-77.2	-57.0	-6.33	-6	-6.29	-66.2	-57.0
$\lambda = 2$	-11.26	-6	-8.58	-263.0	-129.0	-6.79	-6	-6.65	-158.0	-129.0
E_g^2	-6.88	-6	-6.78	-44.1	-39.0	-6.22	-6	-6.20	-44.8	-39.0
			$n = 800$			($b_\Delta^* = b_\Delta$)				
$\lambda = 0$ (G^2)	-5.96	-6	-6.03	29.7	-21.0	(-21.0)				
$\lambda = 2/3$ (C-R)	-6.00	-6	-6.05	3.4	-41.0	(-41.0)				
$\lambda = 1$ (X^2)	-6.02	-6	-6.07	-16.0	-57.0	(-57.0)				
$\lambda = 2$	-6.13	-6	-6.16	-100.7	-129.0	(-129.0)				
E_g^2	-6.00	-6	-6.05	2.8	-39.0	(-39.0)				

Note. n = the number of observations, S.B. = simulated bias, A.B. = asymptotic bias $= -2q$, H.A.B. = $b + n^{-1}b_\Delta = -2q + n^{-1}b_\Delta$, S. b_Δ = simulated $b_\Delta = n(\text{S.B.} + 2q)$, G^2 = the log-likelihood ratio statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic, E_g^2 = Eguchi's divergence. The number for model identification is the number of independent parameters.

Table S2 (Corrected after publication for H.A.B., b_Δ and b^*_Δ except the case of $\lambda = 0$ (G^2); May 16, 2019). Simulated and asymptotic biases of the power divergences (the number of replications = 10,000)

The parameter estimators by $\lambda = 1$ (X^2) are used for all power divergences.

The genetics of plants (Fisher, 1970; 4 categories)

Model 1	S.B.	A.B.	H.A.B.	S.b_Δ	b_Δ	S.B.	A.B.	H.A.B.	S.b_Δ	b_Δ
			$n = 50$					$n = 200$		
$\lambda = 0$ (G^2)	-2.31	-2	-2.08	-15.3	-4.1	-2.05	-2	-2.02	-9.9	-4.1
$\lambda = 2/3$ (C-R)	-2.48	-2	-2.21	-24.0	-10.5	-2.09	-2	-2.05	-17.2	-10.5
$\lambda = 1$ (X^2)	-2.64	-2	-2.32	-31.9	-15.9	-2.12	-2	-2.08	-23.2	-15.9
$\lambda = 2$	-3.56	-2	-2.82	-78.1	-41.1	-2.26	-2	-2.21	-51.7	-41.1
E_g^2	-2.44	-2	-2.18	-21.9	-9.2	-2.08	-2	-2.05	-16.2	-9.2
			$n = 800$			(b^*_Δ)				
$\lambda = 0$ (G^2)	-2.00	-2	-2.01	2.6	-4.1	(-7.0)				
$\lambda = 2/3$ (C-R)	-2.01	-2	-2.01	-4.8	-10.5	(-13.7)				
$\lambda = 1$ (X^2)	-2.01	-2	-2.02	-10.8	-15.9	(-19.0)				
$\lambda = 2$	-2.05	-2	-2.05	-38.1	-41.1	(-43.0)				
E_g^2	-2.00	-2	-2.01	-3.9	-9.2	(-13.0)				
Model 2			$n = 50$					$n = 200$		
$\lambda = 0$ (G^2)	-3.77	-4	-4.19	11.5	-9.7	-4.02	-4	-4.05	-3.9	-9.7
$\lambda = 2/3$ (C-R)	-3.85	-4	-4.46	7.3	-23.1	-4.08	-4	-4.12	-15.5	-23.1
$\lambda = 1$ (X^2)	-3.97	-4	-4.68	1.7	-34.0	-4.13	-4	-4.17	-25.7	-34.0
$\lambda = 2$	-4.64	-4	-5.66	-31.9	-83.0	-4.38	-4	-4.42	-75.3	-83.0
E_g^2	-3.75	-4	-4.43	12.4	-21.7	-4.06	-4	-4.11	-12.9	-21.7
			$n = 800$			(b^*_Δ)				
$\lambda = 0$ (G^2)	-4.03	-4	-4.01	-27.0	-9.7	(-14.0)				
$\lambda = 2/3$ (C-R)	-4.05	-4	-4.03	-42.8	-23.1	(-27.3)				
$\lambda = 1$ (X^2)	-4.07	-4	-4.04	-55.1	-34.0	(-38.0)				
$\lambda = 2$	-4.14	-4	-4.10	-110.3	-83.0	(-86.0)				
E_g^2	-4.05	-4	-4.03	-42.0	-21.7	(-26.0)				

Table S2 (Corrected after publication for H.A.B., b_Δ and b_Δ^* except the case of $\lambda = 0$ (G^2); May 16, 2019). (continued)

The parameter estimators by $\lambda = 1$ (X^2) are used for all power divergences.										
The genetics of plants (Fisher, 1970; 4 categories)										
Model 3	S.B.	A.B.	H.A.B.	S. b_Δ	b_Δ	S.B.	A.B.	H.A.B.	S. b_Δ	b_Δ
			$n = 50$					$n = 200$		
$\lambda = 0$ (G^2)	-6.08	-6	-6.42	-4.0	-21.0	-6.04	-6	-6.11	-7.7	-21.0
$\lambda = 2/3$ (C-R)	-6.43	-6	-6.82	-21.3	-41.0	-6.13	-6	-6.21	-25.8	-41.0
$\lambda = 1$ (X^2)	-6.78	-6	-7.14	-39.2	-57.0	-6.21	-6	-6.29	-41.5	-57.0
$\lambda = 2$	-9.18	-6	-8.58	-159.1	-129.0	-6.59	-6	-6.65	-118.0	-129.0
E_g^2	-6.28	-6	-6.78	-14.1	-39.0	-6.11	-6	-6.20	-22.1	-39.0
			$n = 800$			($b_\Delta^* = b_\Delta$)				
$\lambda = 0$ (G^2)	-6.08	-6	-6.03	-66.6	-21.0	(-21.0)				
$\lambda = 2/3$ (C-R)	-6.11	-6	-6.05	-88.1	-41.0	(-41.0)				
$\lambda = 1$ (X^2)	-6.13	-6	-6.07	-105.4	-57.0	(-57.0)				
$\lambda = 2$	-6.23	-6	-6.16	-183.7	-129.0	(-129.0)				
E_g^2	-6.11	-6	-6.05	-86.1	-39.0	(-39.0)				

Note. n = the number of observations, S.B. = simulated bias, A.B. = asymptotic bias $= -2q$, H.A.B. = $b + n^{-1}b_\Delta = -2q + n^{-1}b_\Delta$, S. b_Δ = simulated $b_\Delta = n(\text{S.B.} + 2q)$, G^2 = the log-likelihood ratio statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic, E_g^2 = Eguchi's divergence. The number for model identification is the number of independent parameters.

Table S3 (Corrected after publication for H.A.B., b_Δ and b_Δ^*; May 16, 2019). Simulated and asymptotic biases of the power divergences (the number of replications = 10,000)

The parameter estimators by E_g^2 (Eguchi) are used for all power divergences.

The genetics of plants (Fisher, 1970; 4 categories)

Model 1	S.B.	A.B.	H.A.B.	S. b_Δ	b_Δ	S.B.	A.B.	H.A.B.	S. b_Δ	b_Δ
	n = 50					n = 200				
$\lambda = 0$ (G^2)	-2.24	-2	-2.08	-12.0	-4.1	-2.04	-2	-2.02	-8.9	-4.1
$\lambda = 2/3$ (C-R)	-2.44	-2	-2.21	-21.9	-10.5	-2.07	-2	-2.05	-14.9	-10.5
$\lambda = 1$ (X^2)	-2.60	-2	-2.32	-30.0	-15.9	-2.10	-2	-2.08	-20.2	-15.9
$\lambda = 2$	-3.45	-2	-2.82	-72.3	-41.1	-2.23	-2	-2.21	-46.5	-41.1
E_g^2	-2.43	-2	-2.18	-21.4	-9.2	-2.07	-2	-2.05	-13.2	-9.2
	n = 800					(b_Δ^*)				
$\lambda = 0$ (G^2)	-2.00	-2	-2.01	-2.4	-4.1	(-7.0)				
$\lambda = 2/3$ (C-R)	-2.01	-2	-2.01	-8.3	-10.5	(-13.7)				
$\lambda = 1$ (X^2)	-2.02	-2	-2.02	-13.5	-15.9	(-19.0)				
$\lambda = 2$	-2.05	-2	-2.05	-38.1	-41.1	(-43.0)				
E_g^2	-2.01	-2	-2.01	-6.8	-9.2	(-13.0)				
Model 2	n = 50					n = 200				
$\lambda = 0$ (G^2)	-3.68	-4	-4.19	15.8	-9.7	-3.98	-4	-4.05	3.5	-9.7
$\lambda = 2/3$ (C-R)	-3.78	-4	-4.46	11.2	-23.1	-4.02	-4	-4.12	-4.1	-23.1
$\lambda = 1$ (X^2)	-3.89	-4	-4.68	5.7	-34.0	-4.06	-4	-4.17	-11.9	-34.0
$\lambda = 2$	-4.52	-4	-5.66	-25.9	-83.0	-4.26	-4	-4.42	-52.8	-83.0
E_g^2	-3.69	-4	-4.43	15.3	-21.7	-4.00	-4	-4.11	0.2	-21.7
	n = 800					(b_Δ^*)				
$\lambda = 0$ (G^2)	-3.95	-4	-4.01	40.5	-9.7	(-14.0)				
$\lambda = 2/3$ (C-R)	-3.97	-4	-4.03	26.4	-23.1	(-27.3)				
$\lambda = 1$ (X^2)	-3.98	-4	-4.04	15.1	-34.0	(-38.0)				
$\lambda = 2$	-4.05	-4	-4.10	-36.3	-83.0	(-86.0)				
E_g^2	-3.97	-4	-4.03	27.7	-21.7	(-26.0)				

Table S3 (Corrected after publication for H.A.B., b_Δ and b^*_Δ except the case of $\lambda = 0$ (G^2); May 16, 2019). (continued)

The parameter estimators by E^2_g (Eguchi) are used for all power divergences.

The genetics of plants (Fisher, 1970; 4 categories)

Model 3	S.B.	A.B.	H.A.B.	S. b_Δ	b_Δ	S.B.	A.B.	H.A.B.	S. b_Δ	b_Δ
			n = 50					n = 200		
$\lambda = 0$ (G^2)	-6.03	-6	-6.42	-1.5	-21.0	-6.06	-6	-6.11	-13.0	-21.0
$\lambda = 2/3$ (C-R)	-6.43	-6	-6.82	-21.6	-41.0	-6.15	-6	-6.21	-29.8	-41.0
$\lambda = 1$ (X^2)	-6.84	-6	-7.14	-42.0	-57.0	-6.22	-6	-6.29	-44.7	-57.0
$\lambda = 2$	-9.75	-6	-8.58	-187.4	-129.0	-6.59	-6	-6.65	-118.7	-129.0
E^2_g	-6.30	-6	-6.78	-14.8	-39.0	-6.13	-6	-6.20	-25.4	-39.0
			n = 800			($b^*_\Delta = b_\Delta$)				
$\lambda = 0$ (G^2)	-5.91	-6	-6.03	75.2	-21.0	(-21.0)				
$\lambda = 2/3$ (C-R)	-5.93	-6	-6.05	54.3	-41.0	(-41.0)				
$\lambda = 1$ (X^2)	-5.95	-6	-6.07	37.6	-57.0	(-57.0)				
$\lambda = 2$	-6.05	-6	-6.16	-37.7	-129.0	(-129.0)				
E^2_g	-5.93	-6	-6.05	56.0	-39.0	(-39.0)				

Note. n = the number of observations, S.B. = simulated bias, A.B. = asymptotic bias $= -2q$, H.A.B. = $b + n^{-1}b_\Delta = -2q + n^{-1}b_\Delta$, S. b_Δ = simulated $b_\Delta = n(\text{S.B.} + 2q)$, G^2 = the log-likelihood ratio statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic, E^2_g = Eguchi's divergence. The number for model identification is the number of independent parameters.

Table S4 (Corrected after publication for H.A.B., b_Δ and b_Δ^* except the case of λ = 0 (G^2) in Model 2; May 16, 2019). Simulated and asymptotic biases of the power divergences (the number of replications = 10,000)

The parameter estimators by E_g^2 (Eguchi) are used for all power divergences.

3-category truncated Poisson variate (Bishop et al., 1975, p.503)

Model 1	S.B.	A.B.	H.A.B.	S.b_Δ	b_Δ	S.B.	A.B.	H.A.B.	S.b_Δ	b_Δ
			n = 50					n = 200		
λ = 0 (G^2)	-2.00	-2	-1.96	0.1	2.1	-2.01	-2	-1.99	-1.2	2.1
λ = 2/3 (C-R)	-2.03	-2	-1.99	-1.6	0.5	-2.01	-2	-2.00	-2.3	0.5
λ = 1 (X^2)	-2.07	-2	-2.02	-3.3	-1.1	-2.02	-2	-2.01	-3.7	-1.1
λ = 2	-2.25	-2	-2.18	-12.4	-8.9	-2.06	-2	-2.04	-11.3	-8.9
E_g^2	-2.02	-2	-1.97	-0.8	1.3	-2.01	-2	-1.99	-1.3	1.3
			n = 800			(b_Δ^*)				
λ = 0 (G^2)	-1.99	-2	-2.00	4.3	2.1	(-3.1)				
λ = 2/3 (C-R)	-2.00	-2	-2.00	2.9	0.5	(-5.5)				
λ = 1 (X^2)	-2.00	-2	-2.00	1.4	-1.1	(-7.3)				
λ = 2	-2.01	-2	-2.01	-6.3	-8.9	(-15.8)				
E_g^2	-2.00	-2	-2.00	3.8	1.3	(-5.2)				
Model 2			n = 50					n = 200		
λ = 0 (G^2)	-4.17	-4	-4.12	-8.3	-6.2	-4.03	-4	-4.03	-6.1	-6.2
λ = 2/3 (C-R)	-4.29	-4	-4.22	-14.7	-10.9	-4.06	-4	-4.05	-11.3	-10.9
λ = 1 (X^2)	-4.40	-4	-4.29	-19.9	-14.7	-4.08	-4	-4.07	-15.5	-14.7
λ = 2	-4.91	-4	-4.63	-45.6	-31.5	-4.17	-4	-4.16	-34.5	-31.5
E_g^2	-4.28	-4	-4.21	-14.2	-10.4	-4.05	-4	-4.05	-10.9	-10.4
			n = 800			(b_Δ^*)				
λ = 0 (G^2)	-4.01	-4	-4.01	-4.2	-6.2	(-6.2)				
λ = 2/3 (C-R)	-4.01	-4	-4.01	-8.1	-10.9	(-10.9)				
λ = 1 (X^2)	-4.01	-4	-4.02	-11.5	-14.7	(-14.7)				
λ = 2	-4.03	-4	-4.04	-27.4	-31.5	(-31.5)				
E_g^2	-4.01	-4	-4.01	-7.3	-10.4	(-10.4)				

Note. n = the number of observations, S.B. = simulated bias, A.B. = asymptotic bias = $-2q$, H.A.B. = $b + n^{-1}b_\Delta = -2q + n^{-1}b_\Delta$, S.$b_\Delta$ = simulated $b_\Delta = n(\text{S.B.} + 2q)$, G^2 = the log-likelihood ratio statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic, E_g^2 = Eguchi's divergence. The number for model identification is the number of independent parameters.

Table S5 (Corrected after publication for H.A.B., b_Δ and b_Δ^*; May 16, 2019). Simulated and asymptotic biases of the power divergences (the number of replications = 10,000)

The parameter estimators by E_g^2 (Eguchi) are used for all power divergences.

4-category truncated Poisson variate

Model 1	S.B.	A.B.	H.A.B.	S. b_Δ	b_Δ	S.B.	A.B.	H.A.B.	S. b_Δ	b_Δ
			$n = 50$					$n = 200$		
$\lambda = 0$ (G^2)	-2.06	-2	-1.91	-2.8	4.5	-2.00	-2	-1.98	-0.7	4.5
$\lambda = 2/3$ (C-R)	-2.11	-2	-1.96	-5.5	2.1	-2.01	-2	-1.99	-2.6	2.1
$\lambda = 1$ (X^2)	-2.17	-2	-2.01	-8.3	-0.4	-2.02	-2	-2.00	-4.9	-0.4
$\lambda = 2$	-2.45	-2	-2.26	-22.6	-12.8	-2.08	-2	-2.06	-16.8	-12.8
E_g^2	-2.09	-2	-1.93	-4.3	3.4	-2.01	-2	-1.98	-1.0	3.4
			$n = 800$			(b_Δ^*)				
$\lambda = 0$ (G^2)	-1.93	-2	-1.99	58.3	4.5	(-4.2)				
$\lambda = 2/3$ (C-R)	-1.93	-2	-2.00	55.5	2.1	(-7.8)				
$\lambda = 1$ (X^2)	-1.93	-2	-2.00	53.0	-0.4	(-10.7)				
$\lambda = 2$	-1.95	-2	-2.02	40.5	-12.8	(-23.6)				
E_g^2	-1.93	-2	-2.00	56.5	3.4	(-7.5)				
Model 2			$n = 50$					$n = 200$		
$\lambda = 0$ (G^2)	-3.89	-4	-4.10	5.6	-5.2	-4.02	-4	-4.03	-3.3	-5.2
$\lambda = 2/3$ (C-R)	-3.99	-4	-4.26	0.4	-12.8	-4.05	-4	-4.06	-9.9	-12.8
$\lambda = 1$ (X^2)	-4.09	-4	-4.38	-4.6	-18.9	-4.08	-4	-4.09	-15.6	-18.9
$\lambda = 2$	-4.62	-4	-4.93	-31.0	-46.7	-4.22	-4	-4.23	-43.6	-46.7
E_g^2	-3.95	-4	-4.24	2.6	-11.8	-4.04	-4	-4.06	-8.2	-11.8
			$n = 800$			(b_Δ^*)				
$\lambda = 0$ (G^2)	-3.90	-4	-4.01	78.1	-5.2	(-8.5)				
$\lambda = 2/3$ (C-R)	-3.91	-4	-4.02	71.1	-12.8	(-15.6)				
$\lambda = 1$ (X^2)	-3.92	-4	-4.02	65.4	-18.9	(-21.4)				
$\lambda = 2$	-3.95	-4	-4.06	38.9	-46.7	(-47.2)				
E_g^2	-3.91	-4	-4.01	72.2	-11.8	(-14.9)				

Table S5 (Corrected after publication for H.A.B., b_Δ and b_Δ^* except the case of λ = 0 (G^2). (continued)

The parameter estimators by E_g^2 (Eguchi) are used for all power divergences.										
4-category truncated Poisson variate										
Model 3	S.B.	A.B.	H.A.B.	S.b_Δ	b_Δ	S.B.	A.B.	H.A.B.	S.b_Δ	b_Δ
			n = 50					n = 200		
$\lambda = 0$ (G^2)	-6.01	-6	-6.25	-0.4	-12.7	-6.00	-6	-6.06	0.03	-12.7
$\lambda = 2/3$ (C-R)	-6.22	-6	-6.47	-10.8	-23.4	-6.05	-6	-6.12	-10.3	-23.4
$\lambda = 1$ (X^2)	-6.40	-6	-6.64	-20.2	-32.1	-6.09	-6	-6.16	-19.0	-32.1
$\lambda = 2$	-7.41	-6	-7.42	-70.6	-70.8	-6.30	-6	-6.35	-59.6	-70.8
E_g^2	-6.17	-6	-6.45	-8.4	-22.4	-6.04	-6	-6.11	-8.8	-22.4
			n = 800			($b_\Delta^* = b_\Delta$)				
$\lambda = 0$ (G^2)	-5.92	-6	-6.02	64.1	-12.7	(-12.7)				
$\lambda = 2/3$ (C-R)	-5.93	-6	-6.03	52.1	-23.4	(-23.4)				
$\lambda = 1$ (X^2)	-5.95	-6	-6.04	43.0	-32.1	(-32.1)				
$\lambda = 2$	-6.00	-6	-6.09	2.6	-70.8	(-70.8)				
E_g^2	-5.93	-6	-6.03	52.5	-22.4	(-22.4)				

Note. n = the number of observations, S.B. = simulated bias, A.B. = asymptotic bias $= -2q$, H.A.B. = $b + n^{-1}b_\Delta = -2q + n^{-1}b_\Delta$, S.$b_\Delta$ = simulated $b_\Delta = n(\text{S.B.} + 2q)$, G^2 = the log-likelihood ratio statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic, E_g^2 = Eguchi's divergence. The number for model identification is the number of independent parameters.