Analytic Number Theory and Related Topics*

Date : October 12 (Tue) 10:00 – October 15 (Fri) 16:50, 2021
Place : Online via Zoom
Organizers : Hirotaka Akatsuka (Otaru University of Commerce)
Yoshinori Yamasaki (Ehime University)

Abstracts

October 12 (Tue)

10:00 – 10:10 Opening

10:10 – 10:40 **Genki Shibukawa** (Kobe University)

"Artin-style characterizations for multiple gamma and sine functions"

There are many characterizations of the gamma function and their variations (q, multiple, elliptic, p-adic, etc.). In particular, Bohr-Mollerup-type theorems using the equation of functions and logarithmic convexity are well-known. E. Artin gave some other-type characterizations of the gamma function in his famous book. We give a generalization of Artin's results to the case of Barnes-type multiple gamma functions and characterize not only the multiple gamma functions but also Kurokawa-type multiple sine functions.

11:00 – 11:30 **Masato Kobayashi** (Kanagawa University)

"Three new integral representations for Apery constant"
(joint work with Shunji Sasaki)

In this talk, I would like to show new representations with the inverse sine function for the Apery constant and even the Catalan which I found in 2021 July. The proof is a reformulation of work of Ewell (1990) and Yue-Williams (1993) with Wallis integral. Moreover, I will show some integral representations of local dilogarithm and trilogarithm functions as consequences.

13:30 – 14:20 **Takao Komatsu** (Zhejiang Sci-Tech University)

"Bernoulli and Cauchy numbers with level 2 associated with Stirling numbers with level 2"

We introduce poly-Bernoulli numbers and poly-Cauchy numbers with level 2, related to the Stirling numbers of the second kind with level 2, and study several properties of poly-Bernoulli numbers and poly-Cauchy numbers with level 2 from their expressions, relations, and congruences. We also show that poly-Bernoulli numbers with level 2 have strong connections with poly-Cauchy numbers with level 2. We also have some extensions of these numbers to higher level.

* This workshop is partially supported by RIMS and JSPS KAKENHI Grant Number 19K03392.

14:40 – 15:30 **Takumi Noda** (Nihon University)

"Asymptotics for Dirichlet-Hurwitz-Lerch type Eisenstein series and applications"
(joint work with Masanori Katsurada)

Let $s = \sigma + it \in \mathbb{C}$ be a variable, $\alpha, \beta, \mu, \nu \in \mathbb{R}$ parameters, χ and ψ any primitive Dirichlet characters modulo $f(\geq 1)$ and $g(\geq 1)$ respectively, and write $e(s) = e^{2\pi is}$, $e_h(s) = e(s/h) = e^{2\pi is/h}$ for any integer $h \geq 1$, and $\chi_a(m) = \chi(a+m)$ and $\psi_b(n) = \psi(b+n)$ for any $a, b, m, n \in \mathbb{Z}$. We introduce the generalized Eisenstein series $F^{\pm}_{\mathbb{Z}^2}(s; \alpha, \beta, \mu, \nu; \chi_a, \psi_b; z)$, defined for any $z = e^{\pi i/2}\tau \in \mathfrak{H}^+$ (the complex upper half-plane) with $|\arg\tau| < \pi/2$ by

$$F^{\pm}_{\mathbb{Z}^2}(s; \alpha, \beta; \mu, \nu; \chi_a, \psi_b; z) = \sum_{m,n=-\infty}^{\infty}{}' \frac{e_f\{(\alpha+m)\mu\}e_g\{(\beta+n)\nu\}\chi_a(m)\psi_b(n)}{\{\alpha+m+(\beta+n)z\}^s},$$

converging absolutely for $\sigma > 2$. The primed summation symbol indicate omission of the impossible term of the form $1/0^s$, and the argument of each summand is chosen with $\arg\{\alpha+m+(\beta+n)z\} \in [-\pi, \pi[$ in $F^{-}_{\mathbb{Z}^2}$, while $\arg\{\alpha+m+(\beta+n)z\} \in]-\pi, \pi]$ in $F^{+}_{\mathbb{Z}^2}$. The main object of this talk is the arithmetical mean

$$F_{\mathbb{Z}^2}(s; \alpha, \beta; \mu, \nu; \chi_a, \psi_b; z) = \frac{1}{2}\bigl\{F^{-}_{\mathbb{Z}^2}(s; \alpha, \beta; \mu, \nu; \chi_a, \psi_b; z) + F^{+}_{\mathbb{Z}^2}(s; \alpha, \beta; \mu, \nu; \chi_a, \psi_b; z)\bigr\}.$$

We shall show a transformation formula in terms of a certain double q-series, which reduces to a generalized Lambert series, and establish a complete asymptotic expansion for $F_{\mathbb{Z}^2}(s; \alpha, \beta; \mu, \nu; \chi_a, \psi_b; z)$ in the ascending order of τ as $\tau \to 0$. We shall apply them to derive a generalization of a classical Ramanujan's formula for specific values of the Riemann zeta-function, as well as certain representations of Weierstraß' elliptic functions in terms of generalized Lambert series.

15:50 – 16:40 **Seiji Kuga** (Kyushu University)

"A resolvent trace formula of Jacquet–Zagier type for Hilbert Maass forms"

Zagier found a generalized Eichler-Selberg trace formula involving symmetric square L-functions by means of Rankin-Selberg method in computing the trace formula of Hecke operators of elliptic cusp forms. Moreover, Sugiyama and Tsuzuki generalized Zagier's formula for Hilbert modular forms with square-free levels in adelic setting and proved a non-vanishing property of symmetric square L-functions. In this talk, we give an analogy of Sugiyama-Tsuzuki's trace formula for Hilbert Maass forms by using the resolvent kernel function of the Laplace operator as the test function at infinite places.

October 13 (Wed)

9:40 – 10:30 **Takashi Nakamura** (Tokyo University of Science)

"Rapidly convergent series representations of symmetric Tornheim double zeta functions"

In this talk, for $s, t, u \in \mathbb{C}$, we show rapidly (or globally) convergent series representations of the Tornheim double zeta function $T(s,t,u)$ and (desingularized) symmetric Tornheim double zeta functions. As a corollary, we give a new a proof of known results on the values of $T(s,s,s)$ at non-positive integers and the location of the poles of $T(s,s,s)$. Furthermore, we prove that the function $T(s,s,s)$ can not be written by a polynomial in the form of $\sum_{k=1}^{j} c_k \prod_{r=1}^{q} \zeta^{d_{kr}}(a_{kr}s + b_{kr})$, where $a_{kr}, b_{kr}, c_k \in \mathbb{C}$ and $d_{kr} \in \mathbb{Z}_{\geq 0}$.

10:50 – 11:40 **Toshiki Matsusaka** (Nagoya University)

"Stephan's observation on the central binomial series"
(joint work with Beáta Bényi)

The central binomial series is defined by

$$\zeta_{CB}(s) = \sum_{n=1}^{\infty} \frac{1}{n^s \binom{2n}{n}}.$$

For each integer $k \leq 0$, the value $\zeta_{CB}(k)$ is a $\mathbb{Q}$-linear combination of 1 and $\pi/\sqrt{3}$. In 2004, Stephan observed that the rational part is closely related to the poly-Bernoulli numbers. In this talk, we introduce a combinatorial polynomial called the r-Eulerian polynomial, and give a proof of Stephan's observation.

13:30 – 14:20 **Wataru Takeda** (Tokyo University of Science)

"Pieri type formula for the Schur multiple zeta functions"
(joint work with Maki Nakasuji)

The Pieri formula, which is an important formula in both representation and combinatorial theories, decomposes the product of a Schur function and a symmetric polynomial as a linear combination of some related Schur functions. Since the Schur multiple zeta functions (SMZFs, for short) have a similar construction of Schur functions, it is expected that the Pieri formula for SMZFs holds. In this talk, we will introduce a certain extended Jacobi-Trudi formula for SMZFs. The Jacobi-Trudi formula for SMZFs is the determinant formula in terms of the multiple zeta(star) functions, and "extended" means the formula without any assumptions compared with the known result which is with some assumptions. Using this formula, we will give the Pieri formula for Schur multiple zeta functions of hook type. This is joint work with Maki Nakasuji(Sophia University).

14:40 – 15:30 **Kota Saito** (Nagoya University)

"Prime-representing functions and Hausdorff dimensions"

Let $c \geq 2$ be any fixed real number. Matomäki investigated the set of $A > 1$ such that the integer part of A^{c^k} is a prime number for every $k \in \mathbb{N}$. She proved that the set is uncountable, nowhere dense, and has Lebesgue measure 0. In this talk, we show that the set has Hausdorff dimension 1.

15:50 – 16:40 **Ade Irma Suriajaya** (Kyushu University)

"The average number of Goldbach representations, pair correlation of zeros of the Riemann zeta function and error term of the prime number theorem"
(joint work with Daniel A. Goldston)

The Goldbach Conjecture is a long unsolved problem in number theory which considers the possibility of every even integer greater than 2 be written as the sum of two prime numbers. This can be stated in a quantitative form using a counting function of the number of such representations which then reformulates the Goldbach Conjecture as the counting function being always positive for any even integer greater than 2. In most analysis, however, it is convenient to consider such counting function weighted by the von Mangoldt function, and we call the corresponding representations, Goldbach representations. In 1991, Fujii proved under the assumption of the Riemann Hypothesis an estimate of the average number of Goldbach representations of integers up to some given positive number. The error term was improved by Bhowmik and Schlage-Puchta in 2010 and further by Languasco and Zaccagnini in 2012. In this talk, we show an explicit formula for the Fujii-type average number of Goldbach representations which allows us to rewrite the error term in connection with Montgomery's pair correlation function of zeros of the Riemann zeta function. This leads to a further improvement of Fujii's error term and the prime number theorem. This is a joint work with Daniel Goldston from San Jose State University.

October 14 (Thu)

9:40 – 10:30 **Hirotaka Kobayashi** (Nagoya University)

"On the discrete mean of the higher derivative of Hardy's Z-function"

In 1990, Yıldırım obtained an asymptotic formula of the discrete mean of the square of Hardy's Z-function over the zeros of its higher derivative. His result implies an interesting property of Hardy's Z-function and its higher derivative. In this talk, we give a generalization of his formula, an approximate formula of the discrete mean of the square of the j-th derivative of Hardy's Z-function over the zero of the k-th derivative of Hardy's Z-function. In addition, we will talk about the brief history of the higher derivative of Hardy's Z-function and some future works.

10:50 – 11:40 **Kenta Endo** (Nagoya University)

"Generalization of the effectively refined multi-dimensional denseness theorem to the Selberg class"

In 1989, Voronin refined the multidimensional denseness theorem for the Riemann zeta-function to an effective form. After that, Garunkštis, Laurinčikas, Matsumoto, J. & R. Steuding used this theorem to obtain an effective universality-type theorem. In this talk, we generalize these theorems to an element of the Selberg class satisfying some conditions.

13:30 – 14:20 **Masahiro Mine** (Sophia University)

"Extreme value distributions for iterated integrals of the logarithm of the Riemann zeta-function"
(joint work with Kenta Endo and Shōta Inoue)

The growth of the Riemann zeta-function in the critical strip has been studied by many researchers in analytic number theory. The Lindelöf Hypothesis asserts an upper bound for this, which is related to some properties of prime numbers, zeros of $\zeta(s)$, and so on. On the other hand, the distribution of extremely large values of $\log|\zeta(s)|$ has been studied in connection with the theory of large deviations of random variables. For example, Lamzouri–Lester–Radziwiłł proved an asymptotic formula to compare the extreme value distribution of $\log|\zeta(s)|$ with that of a certain random Dirichlet series. In this talk, we improve their result and generalize it for functions defined by iterated integrals of $\log\zeta(s)$.

14:40 – 15:30 **Shōta Inoue** (Tokyo Institute of Technology)

"Joint value distribution for L-functions on the critical line"
(joint work with Junxian Li)

Selberg mentioned that distinct primitive L-functions are statistically independent under some suitable assumptions. Later, Bombieri and Hejhal revealed the independence by showing the joint central limit theorem for L-functions. In this talk, we discuss the large deviations for their joint central limit theorem and an application for moments of L-functions.

15:50 – 16:20 **Kazuma Sakurai** (Nagoya University)

"On the zeros of the k-th derivative of the Dirichlet L-functions under the generalized Riemann hypothesis"

Yıldırım, Akatsuka and Suriajaya investigated the distribution of non-real zeros of derivatives of the Dirichlet L-functions, including the number of zeros up to a height T and the distribution of the real part of non-real zeros. In this talk we obtain sharper estimates for the error terms of their results in the case of the k-th derivative of the Dirichlet L-functions, under the truth of the generalized Riemann hypothesis.

October 15 (Fri)

9:40 – 10:30 **Masatoshi Suzuki** (Tokyo Institute of Technology)

"On canonical systems related to the Schur-Cohn test"

The Schur–Cohn test determines the distribution of the roots of a polynomial with respect to the unit circle by the number of sign changes in the determinants of the minors of a particular matrix consisting of the coefficients.

In this talk, an interpretation of the determinants in the Schur-Cohn test from the perspective of the theory of canonical systems will be reported. The interpretation is a result of the resolution of direct and inverse problems for canonical systems related to exponential polynomials of a specific type.

10:50 – 11:40 **Makoto Kawashima** (Nihon University)

"Holonomic series and orthogonal polynomials"

The Legendre polynomials, which is a famous system of orthogonal polynomials, can be constructed as Pade approximants of logarithmic function. In this talk, we briefly explain a new idea to construct orthogonal polynomials as Pade approximants of holonomic series. Using our framework, let us expose new orthogonal polynomials, including a natural generalization of classical ones, for instance Jacobi polynomials, Chebyshev polynomials, Hermite polynomials etc. As an application, let us give a new linear independence criterion of values of certain class of Gauss hypergeometric functions.

13:30 – 14:20 **Hajime Kaneko** (University of Tsukuba)

"On the sum of digits in the binary expansion of the products of integers"
(joint work with Thomas Stoll)

Many mathematicians have studied the properties of the sum of digits in the binary expansion of positive integers. In recent years, there has been progress related to the research of integers whose sum of digits are small. For instance, Corvaja and Zannier proved that there exist at most finitely many odd perfect powers whose sums of digits in the binary expansion are 4. In this talk, we introduce certain Diophantine equation related to the odd integers such that the sum of digits of the products are small. Moreover, we discuss the finiteness of the number of solutions for such equation.

14:40 – 15:30 **Takafumi Miyazaki** (Gunma University)

"Number of solutions to some purely exponential Diophantine equation in three unknowns"
(joint work with István Pink)

For any fixed relatively prime positive integers a, b and c with $a, b, c > 1$, we consider the number of the solutions to the Diophantine equation (1) $a^x + b^y = c^z$ in positive integers x, y and z. On this topic there have been two important developments in recent years. One is due to Yongzhong Hu and Maohua Le (2018–19), who found a gap principle using elementary number theory methods which gives rise to a large gap among three hypothetical solutions. Their work together with a usual application of Baker's theory of linear forms in two logarithms implies a huge absolute and effectively computable constant such that there are at most two solutions to equation (1) whenever at least one of a, b, c exceeds its constant. The other is due to Reese Scott and Robert Styer (2016), who use strictly elementary methods over imaginary quadratic fields to prove that there are at most two solutions to equation (1) whenever c is odd. In this talk, we improve or use the mentioned previous works to prove that equation (1) in general has at most two solutions except for one specific case. This result is definitive in the sense that there are infinitely many cases which allow equation (1) to have (exactly) two solutions. This is a joint work with István Pink (University of Debrecen).

15:50 – 16:40 **Taka-aki Tanaka** (Keio University)

"On power series generated by simpler sequences and having strong algebraic independence properties"
(joint work with Haruki Ide and Kento Toyama)

A power series $\sum_{k=0}^{\infty} c_k z^{e_k}$ with $c_k \neq 0$ is said to be *a lacunary series* if $\lim_{k\to\infty}(e_{k+1} - e_k) = \infty$. Nishioka and the others independently proved the following: Only linear polynomials can appear as algebraic relations, which are represented as irreducible polynomials with algebraic coefficients, among the values of specific lacunary series at nonzero algebraic numbers. In some cases, such linear polynomials can appear only among the values at algebraic numbers $\alpha_1, \ldots, \alpha_r$ whose ratios of some pairs are roots of unity. As a corollary we have the following: The infinite set consisting of all the values of $\sum_{k=0}^{\infty} z^{k!+k}$ and its derivatives of any order, at any nonzero algebraic numbers within the unit circle, is algebraically independent. This result is deduced from the fact that the sequence $\{k! + k\}_{k\geq 0}$ is distributed infinitely to any of congruence classes $a + N\mathbb{Z}$ for all positive integer N and for all $a \in \{0, 1, \ldots, N-1\}$. In fact, for $\sum_{k=0}^{\infty} z^{k!+k}$ no linear relation exists even among the values at algebraic numbers $\alpha_1, \ldots, \alpha_r$ whose ratios of some pairs are roots of unity, and so there is no algebraic relations among the values at any nonzero algebraic numbers.

On the other hand, Loxton and van der Poorten proved that, only linear polynomials can appear as algebraic relations among the values of $\sum_{k=0}^{\infty} z^{d^k}$, where $d \geq 2$ is a fixed integer; however, such a linear relation can appear also among the values at algebraic numbers whose ratios of any pairs are not necessarily roots of unity.

In this talk, we present the following result: The infinite set consisting of all the values of $\sum_{k=0}^{\infty} z^{d^k+k}$ and its derivatives of any order, at any nonzero algebraic numbers within the unit circle, is algebraically independent. This result is proved by using a kind of descent method, which is essentially different from the previous methods.

16:40 – 16:50 Closing