HW #7: Solutions

QUESTIONS FOR REVIEW

8. Assume the marginal cost of production is greater than the average variable cost. Can you determine whether the average variable cost is increasing or decreasing? Explain.

If the average variable cost is increasing (decreasing), then the last unit produced is adding more (less) to total variable cost than the previous units did, on average. Therefore, marginal cost is above (below) average variable cost. In fact, the point where marginal cost exceeds average variable cost is also the point where average variable cost starts to rise.

11. How does a change in the price of one input change the firm’s long-run expansion path?

The expansion path describes the combination of inputs that the firm chooses to minimize cost for every output level. This combination depends on the ratio of input prices: if the price of one input changes, the price ratio also changes. For example, if the price of an input increases, less of the input can be purchased for the same total cost, and the intercept of the isocost line on that input’s axis moves closer to the origin. Also, the slope of the isocost line, the price ratio, changes. As the price ratio changes, the firm substitutes away from the now more expensive input toward the cheaper input. Thus, the expansion path bends toward the axis of the now cheaper input.

EXERCISES

7. The cost of flying a passenger plane from point A to point B is $50,000. The airline flies this route four times per day at 7am, 10am, 1pm, and 4pm. The first and last flights are filled to capacity with 240 people. The second and third flights are only half full. Find the average cost per passenger for each flight. Suppose the airline hires you as a marketing consultant and wants to know which type of customer it should try to attract, the off-peak customer (the middle two flights) or the rush-hour customer (the first and last flights). What advice would you offer?

The average cost per passenger is $50,000/240 for the full flights and $50,000/120 for the half full flights. The airline should focus on attracting more off-peak customers in order to reduce the average cost per passenger on those flights. The average cost per passenger is already minimized for the two peak time flights.

9. The short-run cost function of a company is given by the equation TC=200+55q, where TC is the total cost and q is the total quantity of output, both measured in thousands.

a. What is the company’s fixed cost?

When q = 0, TC = 200, so fixed cost is equal to 200 (or $200,000).

b. If the company produced 100,000 units of goods, what is its average variable cost?

With 100,000 units, q = 100. Variable cost is 55q = (55)(100) = 5500 (or $5,500,000). Average variable cost is \( \frac{TVC}{q} = \frac{5500}{100} = 55 \), or $55,000.

c. What is its marginal cost per unit produced?

With constant average variable cost, marginal cost is equal to average variable cost, $55 (or $55,000).

d. What is its average fixed cost?

At q = 100, average fixed cost is \( \frac{TFC}{q} = \frac{200}{100} = 2 \) or $2,000.
e. Suppose the company borrows money and expands its factory. Its fixed cost rises by $50,000, but its variable cost falls to $45,000 per 1,000 units. The cost of interest (i) also enters into the equation. Each one-point increase in the interest rate raises costs by $3,000. Write the new cost equation.

Fixed cost changes from 200 to 250, measured in thousands. Variable cost decreases from 55 to 45, also measured in thousands. Fixed cost also includes interest charges: 3i. The cost equation is

\[ C = 250 + 45q + 3i. \]

**REVIEW QUESTIONS**

3. In long-run equilibrium, all firms in the industry earn zero economic profit. Why is this true?

The theory of perfect competition explicitly assumes that there are no entry or exit barriers to new participants in an industry. With free entry, positive economic profits induce new entrants. As these firms enter, the supply curve shifts to the right, causing a fall in the equilibrium price of the product. Entry will stop, and equilibrium will be achieved, when economic profits have fallen to zero.

6. At the beginning of the twentieth century, there were many small American automobile manufacturers. At the end of the century, there are only three large ones. Suppose that this situation is not the result of lax federal enforcement of antimonopoly laws. How do you explain the decrease in the number of manufacturers? (Hint: What is the inherent cost structure of the automobile industry?)

Automobile plants are highly capital-intensive. Assuming there have been no impediments to competition, increasing returns to scale can reduce the number of firms in the long run. As firms grow, their costs decrease with increasing returns to scale. Larger firms are able to sell their product for a lower price and push out smaller firms in the long run. Increasing returns may cease at some level of output, leaving more than one firm in the industry.

11. What assumptions are necessary for a market to be perfectly competitive? In light of what you have learned in this chapter, why is each of these assumptions important?

The two primary assumptions of perfect competition are (1) all firms in the industry are price takers, and (2) there is free entry and exit of firms from the market. This chapter discusses how competitive equilibrium is achieved under these assumptions. The first assumption is important because it means that no firm has any market power. Given no firm has market power, firms will produce where price is equal to marginal cost. In the short run, price could equal marginal cost at a quantity where marginal cost is greater than average cost, implying positive economic profits. With free entry and exit, positive economic profits would encourage other firms to enter. This entry exerts downward pressure on price until price is equal to both marginal cost and minimum average cost.

**EXERCISES**

5. Suppose that a competitive firm’s marginal cost of producing output \( q \) is given by \( MC(q) = 3 + 2q \). Assume that the market price of the firm’s product is $9.

a. What level of output will the firm produce?

To maximize profits, the firm should set marginal revenue equal to marginal cost. Given the fact that this firm is operating in a competitive market, the market price it faces is equal to marginal revenue. Thus, the firm should set the market price equal to marginal cost to maximize its profits:

\[ 9 = 3 + 2q, \text{ or } q = 3. \]

b. What is the firm’s producer surplus?

Producer surplus is equal to the area below the market price, i.e., $9.00, and above the marginal cost curve, i.e., \( 3 + 2q \). Because \( MC \) is linear, producer surplus is a triangle with a base equal to $6 (9 - 3 = 6). The height of the triangle is 3, where \( P = MC \). Therefore, producer surplus is

\[ (0.5)(6)(3) = 9. \]
Chapter 7: The Costs of Production

The Costs of Production

Suppose that the average variable cost of the firm is given by $AVC(q) = 3 + q$. Suppose that the firm’s fixed costs are known to be $3$. Will the firm be earning a positive, negative, or zero profit in the short run?

Profit is equal to total revenue minus total cost. Total cost is equal to total variable cost plus fixed cost. Total variable cost is equal to $(AVC(q))q$. Therefore, at $q = 3$,

$$TVC = (3 + 3)(3) = 18.$$  

Fixed cost is equal to $3$. Therefore, total cost equals $TVC$ plus $TFC$, or

$$TC = 18 + 3 = 21.$$  

Total revenue is price times quantity:

$$TR = (9)(3) = 27.$$  

Profit is total revenue minus total cost:

$$\pi = 27 - 21 = 6.$$  

Therefore, the firm is earning positive economic profits. More easily, you might recall that profit equals producer surplus minus fixed cost. Since we found that producer surplus was $9$ in part b, profit equals $9 - 3$ or $6$.

Suppose the cost function is $C(q) = 4q^2 + 16$.

1. Find variable cost, fixed cost, average cost, average variable cost, and average fixed cost. Hint: Marginal cost is $MC=8q$.

Variable cost is that part of total cost that depends on $q$ ($4q^2$) and fixed cost is that part of total cost that does not depend on $q$ ($16$).

$$VC = 4q^2$$  

$$FC = 16$$  

$$AC = \frac{C(q)}{q} = 4q + \frac{16}{q}$$  

$$AVC = \frac{VC}{q} = 4q$$  

$$AFC = \frac{FC}{q} = \frac{16}{q}$$

2. Show the average cost, marginal cost, and average variable cost curves on a graph.
Average cost is u-shaped. Average cost is relatively large at first because the firm is not able to spread the fixed cost over very many units of output. As output increases, average fixed costs will fall relatively rapidly. Average cost will increase at some point because the average fixed cost will become very small and average variable cost is increasing as q increases. Average variable cost will increase because of diminishing returns to the variable factor labor. MC and AVC are linear, and both pass through the origin. Average variable cost is everywhere below average cost. Marginal cost is everywhere above average variable cost. If the average is rising, then the marginal must be above the average. Marginal cost will hit average cost at its minimum point.

3. Find the output that minimizes average cost.

The minimum average cost quantity is where MC is equal to AC:

\[ AC = 4q + \frac{16}{q} = 8q = MC \]

\[ \frac{16}{q} = 4q \]

\[ 16 = 4q^2 \]

\[ q = 2\sqrt{2} \]

4. At what range of prices will the firm produce a positive output?

The firm will supply positive levels of output as long as \( P = MC > AVC \), or as long as the firm is covering its variable costs of production. In this case, marginal cost is everywhere above average variable cost so the firm will supply positive output at any positive price.

5. At what range of prices will the firm earn a negative profit?

The firm will earn negative profit when \( P = MC < AC \), or at any price below minimum average cost. In part c above we found that the minimum average cost quantity was \( q = 2 \). Plug \( q = 2 \) into the average cost function to find \( AC = 16 \). The firm will therefore earn negative profit if price is below 16.

6. At what range of prices will the firm earn a positive profit?

In part e we found that the firm would earn negative profit at any price below 16. The firm therefore earns positive profit as long as price is above 16.

10. Suppose you are given the following information about a particular industry:

\[ Q^D = 6500 - 100P \quad \text{Market demand} \]

\[ QS = 1200P \quad \text{Market supply} \]

\[ C(q) = 722 + \frac{q^2}{200} \quad \text{Firm total cost function} \]

\[ MC(q) = \frac{2q}{200} \quad \text{Firm marginal cost function} \]

Assume that all firms are identical, and that the market is characterized by pure competition.

a. Find the equilibrium price, the equilibrium quantity, the output supplied by the firm, and the profit of the firm.

Equilibrium price and quantity are found by setting market supply equal to market demand, so that 6500-100P=1200P. Solve to find P=5 and substitute into either equation to find Q=6000.

To find the output for the firm set price equal to marginal cost so that \( 5 = \frac{2q}{200} \) and \( q = 500 \).

Profit of the firm is total revenue minus total cost or

\[ \Pi = pq - C(q) = 5(500) - 722 - \frac{500^2}{200} = 528. \]

Notice that since the total output in the market is 6000, and the firm output is 500, there must be 6000/500=12 firms in the industry.
Chapter 7: The Costs of Production

b. Would you expect to see entry into or exit from the industry in the long-run? Explain. What effect will entry or exit have on market equilibrium?

Entry because the firms in the industry are making positive profit. As firms enter, the supply curve for the industry will shift down and to the right and the equilibrium price will fall, all else the same. This will reduce each firm’s profit down to zero until there is no incentive for further entry.

c. What is the lowest price at which each firm would sell its output in the long run? Is profit positive, negative, or zero at this price? Explain.

In the long run the firm will not sell for a price that is below minimum average cost. At any price below minimum average cost, profit is negative and the firm is better off selling its fixed resources and producing zero output. To find the minimum average cost, set marginal cost equal to average cost and solve for q:

\[
\frac{2q}{200} = \frac{722}{q} + \frac{q}{200}
\]

\[
\frac{q}{200} = \frac{722}{q}
\]

\[
q^2 = 722(200)
\]

\[
q = 380
\]

\[
AC(q = 380) = 3.8.
\]

Therefore, the firm will not sell for any price less than 3.8 in the long run.

d. What is the lowest price at which each firm would sell its output in the short run? Is profit positive, negative, or zero at this price? Explain.

The firm will sell for any positive price, because at any positive price marginal cost will be above average variable cost (AVC=q/200). Profit is negative as long as price is below minimum average cost, or as long as price is below 3.8.